

# Loudspeakers in Vented Boxes: Part I\*

A. N. THIELE

*Australian Broadcasting Commission, Sydney, N.S.W. 2001, Australia*

An investigation of the equivalent circuits of loudspeakers in vented boxes shows that it is possible to make the low-frequency acoustic response equivalent to an ideal high-pass filter or as close an approximation as is desired. The simplifying assumptions appear justified in practice and the techniques involved are simple.

The low-frequency performance of a loudspeaker can be adequately defined by three parameters, the resonant frequency  $f_s$ , a volume of air  $V_{as}$ , equivalent to its acoustic compliance, and the ratio of electrical resistance to motional reactance at the resonant frequency  $Q_s$ . From these three parameters, the electroacoustic efficiency  $\eta$  can be found also. A plea is made to loudspeaker manufacturers to publish these parameters as basic information on their product. The influence of other speaker constants on these parameters is investigated.

When  $f_s$  and  $V_{as}$  are known, a loudspeaker box can be designed to give a variety of predictable responses which are different kinds of high-pass 24-dB per octave filters. For each response, a certain value of  $Q$  is required which depends not only on the  $Q_s$  of the loudspeaker but also the damping factor of the amplifier, for which a negative value is often required.

The usual tuning arrangement leads to a response which can be that of a fourth-order Butterworth filter. This, however, is only a special case, and a whole family of responses may be obtained by varying the volume and tuning of the box. Also an empirical "law" is observed that for a given loudspeaker the cutoff frequency depends closely on the inverse square root of the box volume. The limitations of this "law" may be overcome by the use of filtering in the associated amplifier. For example, for a given frequency response, the box volume can be reduced at the price of increased low-frequency output from the amplifier and vice versa, with little change in the motion required of the loudspeaker.

Acoustic damping of the vent is shown to be unnecessary. Examples are given of typical parameters and enclosure designs.

**Editor's Note:** The theory of vented-box or bass-reflex loudspeaker baffles has always seemed to have an air of mystery, probably because the total electroacoustic system has four degrees of freedom and seems four times as complicated as the closed-box baffle with its two degrees of freedom. Beranek gives a good foundation for theoretical analysis and Novak has performed numerous

valuable calculations. Those working in the design of loudspeakers have used these analysis techniques and probably asked essentially the same seven questions that A. N. Thiele recognized at the turn of the previous decade.

The seven questions and their answers were published in the August 1961 issue of the *Proceedings of the IRE Australia*, and the elegance of the answers adequately justifies republication of Thiele's work in the *Journal of the Audio Engineering Society*. In his classic discourse Thiele observes that the topology of the equivalent circuit (Fig. 1) is simply that of a high-pass filter. If suffi-

\* Presented at the 1961 I.R.E. Radio and Electronic Engineering Convention, Sydney, N.S.W., March 1961. Reprinted from *Proceedings of the IRE Australia*, vol. 22, pp. 487-508 (Aug. 1961). The author was formerly with E.M.I. (Aust.) Ltd., Sydney, N.S.W.

cient simplification can be justified, Thiele reasons that the methods of modern network synthesis should be applicable to loudspeakers. This is a profound observation because it means that once the system transfer function is chosen, a logical sequence can be followed to specify driver and baffle parameters. This is much more efficient than the cut and try methods based on either analysis or measurements.

Although the idea is profound because of its simplicity, much work is required to develop, utilize, and demonstrate its use. In the interest of compatibility with format in this Journal, we have received permission from A. N. Thiele to republish his work in two parts. This first part develops the synthesis approach and summarizes all of vented-box design in a table of 28 alignments. The second part will apply the method and draw some very pertinent conclusions about efficiency, driver  $Q$ , box volume, and amplifier output impedance.

The high point of this work is Table I which gives 28 alignments for vented-box loudspeakers. I have been so impressed with this table that I have written a Fortran program to quickly apply Thiele's synthesis methods to any loudspeaker with adequately known parameters. This program and a run or two for typical woofers will be published after Part II.

In considering this manuscript for republication, Thiele has suggested that after 10 years his only change of attitude would be to change the emphasis in Section XIV (Part II). In contrast to the original preference for use of a closed box (which is still quite valid), Thiele would now emphasize the use of a vented box for measurements. This is indeed a trifling matter and in concurring with Thiele's opinion, I can only add emphasis to how well this paper has passed the test of time—it is just as pertinent now as it was ten years ago.

*J. R. Ashley*

**I. INTRODUCTION:** The technique of using a vented box to obtain adequate low-frequency response from a loudspeaker has been known for many years. The principle seems simple, yet the results obtained are variable. Since comparatively cheap and reliable methods of acoustic measurement, especially at low frequencies, virtually do not exist, the only check of results is the "listening test." The listening test is after all the final criterion of the performance of an electroacoustic system, but as a method of adjusting for optimum it is very poor indeed. Quite apart from one's prejudices and memories of previous "acceptable" equipments, the adjustment of a vented box in ignorance of the loudspeaker parameters involves two simultaneous adjustments, box tuning and amplifier damping. And again there is a strong temptation to adjust the low-frequency response to something other than flat to "balance" response errors at high frequencies, when in fact the two problems should be tackled separately.

For a long time it has seemed to the writer that the methods of design of vented boxes were unsatisfactory, leaving a number of questions unanswered.

1) What size of box should be chosen? Usually it seems the larger the better, but how much better is a large box and what penalty does one pay for a small box? And for a given speaker, what is a "large" box or a "small" box?

2) What amplifier damping should be used? In general

the answer is, the heavier the damping the better, though with high-efficiency speakers this could cause a loss of low frequencies. But then again, negative damping is sometimes used, especially in the United States. And when vented enclosures often give excellent results, why should they be known by some as "boom boxes"?

3) Is it advisable or necessary to use acoustic damping to flatten the response? Some claim good results [1] while others [2] warn against it. The general principle of flattening response with parasitic resistance, and thus dissipating hard-won power, seems wrong, especially in an output stage and when a maximum bandwidth is sought. The principle seems to apply equally to an amplifier-loudspeaker-box combination and a video output stage.

4) To what frequency should the vent be tuned? The conventional answer is to tune it to the loudspeaker resonant frequency, but Beranek [3, p. 254] mentions that "for a very large enclosure, it is permissible to tune the port to a frequency below the loudspeaker resonance," while small boxes are sometimes tuned above loudspeaker resonance.

5) What should be the area of the vent? The conventional answer is to make it equal to the piston area of the loudspeaker, but Novak [2] states that "it is permissible to use any value of vent area," and again "the vent area should not be allowed to be less than 4 in<sup>2</sup>." Again, should we use only a hole for the vent or should we use a duct or tunnel?

6) If we equalize the amplifier to correct deficiencies in the speaker and enclosure, what penalties result for example in distortion? Can we trade amplifier size for box size?

7) Assuming that we know how to design a box (and associated amplifier) given the loudspeaker parameters, how may the parameters be measured?

There are other questions that could be asked but the seven above seem the most important; at any rate, they are the ones that the present paper hopes to answer.

## II. DERIVATION OF BASIC THEORY

The theory of operation of loudspeakers in vented boxes has been covered so many times in the literature [3, pp. 208-258], [4] that it should be unnecessary to repeat it here; therefore only sufficient of the theory will be quoted to make the present approach intelligible.

This approach derives from Novak [2] to whom the reader is referred, not only for his method, but for his introductory paragraph . . . "Trade journals tell of 'all new enclosures, revolutionary concepts, and totally new principles of acoustics' when in reality there is a close identity with enclosure systems described long ago in well-known classics on acoustics." This should be framed and hung on the audio engineer's wall alongside Lord Kelvin's dictum. The present paper is the result of a different emphasis on, and interpretation of, Novak's treatment. It should be emphasized that, unless stated specifically otherwise, the results apply only to the "piston range" of the speaker. This is the region where the circumference of the speaker is less than the wavelength of radiated sound, i.e., below 400 Hz for a 12-inch speaker, and below 1 kHz for a 5-inch speaker. The performance of loudspeakers above the piston range is another subject altogether.

We will be dealing later with a simplified equivalent

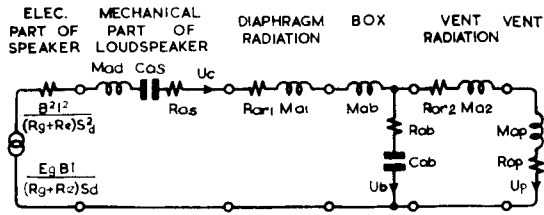


Fig. 1. Complete (electromechanical) acoustical circuit of loudspeaker in vented box (after Beranek [3]).

circuit, but first consider Fig. 1 in which the complete equivalent circuit of the loudspeaker and enclosure is given in acoustical terms.

We note that there are three possible equivalent circuits, electrical, mechanical, and acoustical. To convert from electrical to mechanical units,

$$Z_m = B^2 l^2 / Z_e \quad (1)$$

where

- $Z_e$  electrical impedance
- $Z_m$  equivalent mechanical impedance
- $B$  magnetic flux density in air gap
- $l$  length of wire in air gap.

Again to convert from mechanical to acoustic units,

$$Z_a = Z_m / S_d^2 \quad (2)$$

where

- $Z_a$  acoustical impedance
- $S_d$  equivalent piston area of diaphragm (usually taken as area inside first corrugation).

Taking then in Fig. 1 the first impedance after the generator which is the acoustical equivalent of the electrical resistance of the amplifier output impedance  $R_g$  in series with the voice coil resistance  $R_e$ , we can see that the various equivalents for this impedance are

$$Z_e = R_g + R_e \quad (3)$$

$$Z_m = B^2 l^2 / (R_g + R_e) \quad (4)$$

$$Z_a = B^2 l^2 / S_d^2 (R_g + R_e). \quad (5)$$

In Fig. 1,

- $E_g$  open-circuit voltage of audio amplifier
- $M_{ad}$  ( $= M_{md} / S_d^2$ ) acoustic mass of diaphragm and voice coil
- $M_{md}$  mechanical mass as usually measured
- $C_{as}$  acoustic compliance of suspension
- $R_{as}$  acoustic resistance of suspension
- $R_{ar1}$  acoustic radiation resistance for front side of loudspeaker diaphragm
- $M_{a1}$  acoustic radiation mass (air load) for front side of loudspeaker diaphragm
- $M_{ab}$  acoustic mass of air load on rear side of loudspeaker
- $R_{ab}$  acoustic resistance of box
- $C_{ab}$  acoustic compliance of box
- $R_{ar2}$  acoustic radiation resistance of vent
- $M_{a2}$  acoustic radiation mass (air load) of vent
- $M_{ap}$  acoustic mass of air in vent
- $R_{ap}$  acoustic resistance of air in vent
- $U_c$  volume velocity of cone
- $U_b$  volume velocity of box
- $U_p$  volume velocity of port, or vent.

The advantage of using this large complete equivalent circuit in the first place is that the equivalent circuit of the loudspeaker in a totally enclosed box may be shown by removing the mesh representing the vent. To represent the speaker operated in an infinite baffle,  $C_{ab}$  and  $R_{ab}$  are short-circuited. If the speaker is operated in open air (unbaffled), the circuit is as in an infinite baffle, but the values of  $R_{ar1}$  and  $M_{a1}$  are modified [see 4, Fig. 5.2]. The details of these circuits are very well covered in [3] from which Fig. 1 and the accompanying symbols are taken.

To make the circuit more manageable, we simplify it to Fig. 2.

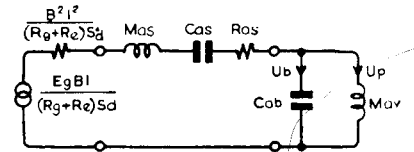


Fig. 2. Simplified acoustical circuit of loudspeaker in vented box.

1) The three acoustic masses  $M_{ad}$ ,  $M_{a1}$ , and  $M_{ab}$  are lumped together to make a single mass  $M_{as}$ . However, we must be careful to remember that this is an artifice.  $M_{as}$  is not fixed, and some error results by assuming it to be so. For example, the reduction of  $M_{ab}$  and hence of  $M_{as}$  when the speaker is tested in open air causes a rise in resonant frequency, which must be accounted for in measurements, as in Section XIV.

2)  $R_{ar1}$  and  $R_{ar2}$  are neglected in the equivalent circuit, even though they are responsible for the acoustic output of the loudspeaker. The whole essence of Novak's theoretical model which makes a simple solution possible is that a loudspeaker is a most inefficient device. In measurements of fifty loudspeakers using the method of Section XIV covering a wide range of sizes and qualities, efficiencies ranged between 0.4% and 4%. For this reason, the radiation resistances may be safely neglected. Since radiation resistance varies with frequency squared, this simplifies analysis considerably. For, as pointed out in [3, p. 216], the radiation resistance of a loudspeaker in a "medium-sized box (less than 8 ft<sup>3</sup>)" is approximately the radiation impedance for a piston in the end of a long tube. And the radiation resistance of the vent (or port) is the same. Thus

$$R_{ar1} = R_{ar2} = \pi f^2 \rho_0 / c \quad (6)$$

where  $\rho_0$  is the density of air and  $c$  is the velocity of sound in air.

Note that the radiation resistance is independent of the dimensions of the piston or vent. Note also that Eq. (6) is an approximation which is accurate only in the piston range of the loudspeaker (compare [3, Fig. 5.7] or [4, Fig. 5.2]).

3)  $M_{a2}$  and  $M_{ap}$  are lumped together as  $M_{av}$ , the total air mass of the vent.

4)  $R_{ab}$  and  $R_{ap}$  are neglected since for most practical purposes their  $Q$  is very high compared with that of the loudspeaker, especially when its damping is properly controlled by the amplifier.

For example, it will be shown later that the  $Q$  of speak-

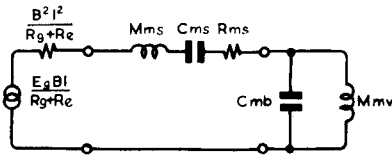


Fig. 3. Simplified mechanical circuit of loudspeaker in vented box.

er plus amplifier for a vented box will usually lie between 0.3 and 0.5. The  $Q$  of the vent, on the other hand, can be found by combining [3, Eqs. (5.54) and (5.55)] to give

$$Q_v = \omega M_{ap} / R_{ap} = (S_v f / \mu)^{1/2} (l' + 1.70a) / (l' + 2a) \quad (7)$$

where

- $Q_v$  effective  $Q$  of vent
- $S_v$  area of vent (assumed to have constant cross section)
- $l'$  actual length of vent
- $a$  effective radius of vent
- $\mu$  kinematic coefficient of viscosity; for air at NTP,  $\mu = 1.56 \times 10^{-5} \text{ m}^2/\text{s}$ .

Thus if  $S_v = 4 \text{ in}^2$ , the bottom limit specified by Novak, and  $f = 25 \text{ Hz}$ , then  $Q_v = 64$ .

Since these are the smallest values of  $S_v$  and  $f$  likely to be found in practice, it is clear that little error will result from this source, and this is confirmed in Section XI. In the preceding discussion, the effect of  $M_{a2}$  and  $R_{ar2}$  has been neglected, but in no case investigated has the total  $Q_v$  fallen below 30.

5) As a result of measurements of fifty loudspeakers, it appears that the  $Q_a$  of the speaker due to  $R_{as}$  lies usually between 3 and 10, so that this does not affect matters greatly, but since  $R_{as}$  can be lumped with the equivalent electrical resistance (see Eq. (8)) and because it has some importance in the loudspeaker measurements of Section XIV, it is included in Fig. 2

The mechanical equivalent circuit (Fig. 3) is derived from Fig. 2 by multiplying all the acoustical impedances by the conversion factor  $S_d^2$  as in Eq. (2). Thus these impedances represent the mechanical impedances at the loudspeaker diaphragm due to the whole acoustical-mechanical circuit. Since the conversion is obtained by multiplying by a constant, the form of the circuit remains the same. However, when the conversion is made from Fig. 3 to Fig. 4, the electrical equivalent circuit, it can be seen from Eq. (1) that an impedance inversion takes place. Thus all series elements become parallel elements, inductances become capacitances, and vice versa. Thus  $L_{ces}$  is the electrical inductance due to the compliance of the loudspeaker suspension,  $C_{mes}$  is the electrical capacitance due to the mass of the loudspeaker cone,  $C_{mev}$  is the electrical capacitance due to the mass of the vent, and  $L_{ceb}$  is the electrical inductance due to the compliance of the box. In Fig. 4 an additional pair of circuit elements which were neglected in the earlier circuits have been added within the dashed lines. These are the inductance and shunt resistance (largely due to eddy current loss in the pole piece and front plate) of the voice coil.

It is hoped that this will not cause confusion. These elements contribute very small effects at the low frequencies we are considering, but show the reason for the

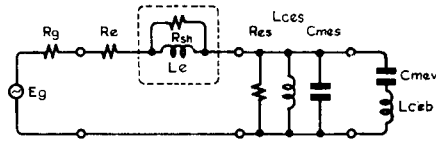


Fig. 4. Simplified electrical circuit of loudspeaker in vented box.

shape of the resulting electrical impedance curve of Fig. 5 above  $f_n$ . However, this will be of greater importance when we come to testing procedures in Section XIV.

### III. DERIVATION OF RESPONSE CURVE

The expression for the frequency response of the system is obtained by analysing the circuit of Fig. 2. To simplify the expression, we lump all the series resistance into a total acoustic resistance,

$$R_{at} = R_{as} + [B^2 l^2 / (R_g + R_e) S_d^2]. \quad (8)$$

Now we have seen already that the radiation resistances of speaker and vent must always be the same. And since the radiated sound depends on the sum of the volume velocities  $U_c$  and  $U_p$  (or rather their difference, since  $U_p$  derives from the back pressure of the speaker), then the acoustic power output is

$$W_{ao} = |U_c - U_p|^2 R_{ar1} \quad (9)$$

while the nominal electrical input power is

$$W_{ei} = E_g^2 R_e / (R_g + R_e)^2. \quad (10)$$

Thus the efficiency is

$$\eta = W_{ao} / W_{ei} = [|U_c - U_p|^2 R_{ar1} (R_g + R_e)^2] / (E_g^2 R_e). \quad (11)$$

Analyzing the circuit, we find that

$$(U_c - U_p) / [E_g Bl / S_d (R_g + R_e)] = 1 / p M_{as} \times \left[ \frac{p^4 M_{as} M_{av} C_{as} C_{ab}}{p^4 M_{as} M_{av} C_{as} C_{ab} + p^3 M_{av} C_{as} C_{ab} R_{at} + p^2 (M_{as} C_{as} + M_{av} C_{as} + M_{av} C_{ab}) + p C_{as} R_{at} + 1} \right]. \quad (12)$$

To make the expression easier to manage we write  $E(p)$  for the expression inside the square bracket on the right-hand side which is a fourth-order high-pass filtering function. Also if  $j\omega$  is written for  $p$ , the steady-state response  $E(j\omega)$  is found. We also convert  $p M_{as}$  from the operational form to the steady-state form  $j\omega M_{as}$ , and then substitute

$$M_{ms} = M_{as} S_d^2. \quad (13)$$

This puts the expression for mass into a more intelligi-

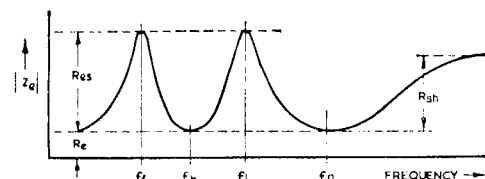


Fig. 5. Typical impedance curve of loudspeaker in vented box.

ble form, but it is emphasized that the total loudspeaker mechanical mass  $M_{ms}$  includes not only the mass of the cone plus voice coil, but also the mechanical equivalent of the acoustic air load. The latter is only a small part of the total, but varies with the speaker's environment, e.g., box volume [3]. Thus if we substitute Eqs. (6), (12), and (13) in Eq. (11),

$$\eta = \rho_0 B^2 l^2 S_d^2 |E(j\omega)|^2 / 4\pi c R_e M_{ms}^2 \quad (14)$$

or

$$\eta = (\rho_0 / 4\pi c) (B^2 l^2 S_d^2 / R_e M_{ms}^2) |E(j\omega)|^2. \quad (15)$$

Thus the expression for efficiency contains three parts:

- 1) a constant part containing physical constants,
- 2) a constant part containing speaker parameters,
- 3) a part  $|E(j\omega)|^2$  which varies with frequency.

#### IV. CONTROLLING THE FREQUENCY RESPONSE

The problem of greatest interest is the control of frequency response; so we consider first (3),  $|E(j\omega)|^2$ , or preferably its operational form  $E(p)$ . To make this easier to manage we substitute in  $E(p)$  of Eq. (12)

$$T_s^2 = (1/\omega_s)^2 = M_{as} C_{as} \quad (16)$$

$$T_b^2 = (1/\omega_b)^2 = M_{ab} C_{ab} \quad (17)$$

$$Q_t = (M_{as}/C_{as})^{1/2} / R_{at} \quad (18)$$

where  $\omega_s$  is the resonant frequency,  $\omega_b$  is the box resonant frequency, or more exactly, the frequency at which the acoustic mass of the vent resonates with the acoustic capacitance of the box. It should not be confused, as is often done, with  $f_h$  or  $f_l$  of Fig. 5, which are by-products of  $f_s$  and  $f_b$  (see Eqs. (105) and (106)).

$Q_t$  is the total  $Q$  of the loudspeaker when connected to its amplifier. The acoustic resistance in the loudspeaker  $R_{as}$  has a small effect, but usually the resistances reflected from the loudspeaker resistance  $R_e$  and the amplifier  $R_g$  contribute the greater part of  $Q_t$ . Then  $E(p)$  of Eq. (12) becomes

$$E(p) =$$

$$\frac{p^4 T_b^2 T_s^2}{\left\{ \frac{p^4 T_b^2 T_s^2 + p^3 (T_b^2 T_s / Q_t)}{+ p^2 [T_b^2 + T_s^2 + T_b^2 C_{as} / C_{ab}] + p (T_s / Q_t) + 1} \right\}} \quad (19)$$

For many purposes this is more conveniently written as

$$E(p) = 1 / \{ 1 + 1/p Q_t T_s + (1/p^2) [1/T_b^2 + 1/T_s^2 + C_{as}/C_{ab} T_s^2] + 1/p^3 T_b^2 T_s Q_t + 1/p^4 T_b^2 T_s^2 \}. \quad (20)$$

This expression corresponds to Novak's expression for the modulus in his Eq. (15) which is simplified into his Eq. (16). (Note that in the captions for his Figs. 7, 9, 11, 12, and 13, a positive sign is wrongly substituted for a negative sign).

As stated before, this is a fourth-order high-pass function, that is, it has an asymptotic slope in the attenuation band of 24 dB per octave, and can be written in the general form

$$E(p) = 1 / \{ 1 + x_1/p T_0 + x_2/p^2 T_0^2 + x_3/p^3 T_0^3 + 1/p^4 T_0^4 \} \quad (21)$$

which is defined by a time constant  $T_0$  ( $= 1/\omega_0$ , the

nominal cutoff frequency) and three coefficients  $x_1, x_2, x_3$  which determine the shape of the response curve. In fact, the general expression is often written with a constant  $x_0$  and  $x_4$  instead of the two unity coefficients in the denominator of Eq. (21); but the expression can always be reduced to the form of Eq. (21) by division of the whole expression by a constant, and suitable adjustment of  $T_0$  and the  $x$  coefficients. Considering Eq. (20) now from the viewpoint of what can be done with a given speaker, the parameters  $C_{as}$  and  $T_s$  are fixed. Thus there are three variables  $Q_t, T_b$ , and  $C_{ab}$ , and it is possible to achieve any desired shape of curve (i.e., any desired combination of the three  $x$  coefficients); but in doing so  $T_0$  is determined (see Eq. (27)).

For identity between the two Eqs. (20) and (21), the coefficients of the various powers of  $p$  must be identical, that is,

$$x_1/T_0 = 1/Q_t T_s \quad (22)$$

$$x_2/T_0^2 = 1/T_b^2 + 1/T_s^2 + C_{as}/C_{ab} T_s^2 \quad (23)$$

$$x_3/T_0^3 = 1/Q_t T_b^2 T_s \quad (24)$$

$$1/T_0^4 = 1/T_b^2 T_s^2. \quad (25)$$

From these, the relationships can be established

$$T_b/T_s = x_1/x_3 \quad (26)$$

$$T_0/T_s = (x_1/x_3)^{1/2} \quad (27)$$

$$Q_t = 1/(x_1 x_3)^{1/2} \quad (28)$$

$$C_{as}/C_{ab} = (x_1 x_2 x_3 - x_3^2 - x_1^2)/x_1^2. \quad (29)$$

The Hurwitz criteria [5] for stability of a network defined by Eq. (21) are

- 1) all the  $x$  coefficients are positive,
- 2)  $x_1 x_2 x_3 - x_3^2 - x_1^2$  is positive.

If (1) and (2) are true, then all the parameters determined by the four Eqs. (26)–(29) are positive and therefore realizable. Thus we have in the four equations a set of simple relationships which enable us to achieve, for any speaker, any shape of low-frequency cutoff (fourth-order) characteristic. The only requirement is that we have sufficient freedom to choose a suitable box resonant frequency  $1/T_b$ , box volume  $C_{ab}$ , and total  $Q$  of speaker plus amplifier  $Q_t$ , and can accept the resulting value of  $T_0$ .

The first parameter  $T_b$  presents no practical difficulty; the second,  $C_{ab}$ , can cause trouble if space is limited, but in this case, as shown in Section VII, we can work backward and choose a suitable response characteristic to suit the box size; the third,  $Q_t$ , is controlled by the source impedance of the amplifier. If the required  $Q_t$  is greater than the speaker's natural  $Q$ , a positive output impedance will be required of the amplifier and this can be controlled by the usual negative feedback techniques. If less, a negative output impedance will be required, and this can be achieved by applying feedback from a separate winding on the voice coil, or by a combination of positive current and negative voltage feedback. There is a practical limit here if the degree of negative impedance required is too large, but this will be discussed in Section XII.

#### V. SOME PRACTICAL RESPONSE CURVE SHAPES

##### Fourth-Order Butterworth Response

Armed with Eqs. (26)–(29) we can calculate the parameters required for different response characteristics. The most obvious one to try first is the fourth-order

maximally flat (Butterworth)<sup>1</sup> characteristic for which

$$|E(j\omega)| = 1/[1 + (\omega_o/\omega)^8]^{1/2} \quad (30)$$

or

$$|E(j\omega)|^2 = 1/[1 + (\omega_o/\omega)^8] \quad (31)$$

and, in the operational form,

$$E(p) = 1/(1 + 2.613/pT_o + 3.414/p^2T_o^2 + 2.613/p^3T_o^3 + 1/p^4T_o^4). \quad (32)$$

Note that in Eq. (31) and others which will follow, the ratio of any two frequencies, say  $\omega_a/\omega_b$ , is identical to  $f_a/f_b$ . Note also that all Butterworth responses are 3 dB down when  $\omega = \omega_o$ , i.e.,  $\omega T_o = 1$ .

A characteristic of Butterworth responses, though not peculiar to them, which simplifies calculations even further is that in all cases

$$x_1 = x_3. \quad (33)$$

Thus in this class or response,

$$T_b = T_s \quad (34)$$

$$T_o = T_s \quad (35)$$

$$Q_t = 1/x_1 \quad (36)$$

$$C_{as}/C_{ab} = x_2 - 2. \quad (37)$$

Thus in the fourth-order case where

$$x_1 = x_3 = 2.613 \quad (38)$$

$$x_2 = 3.414 \quad (39)$$

we have

$$Q_t = 0.383 \quad (40)$$

$$C_{as}/C_{ab} = 1.414. \quad (41)$$

This is alignment no. 5 of Table I. The term "alignment" seems appropriate since the problem is similar to the choice of alignments for other filters, e.g., RF and IF amplifiers. This is obviously the conventional type of box alignment, for the box frequency  $f_b$  is identical with the speaker resonant frequency  $f_s$ , and also the frequency  $f_3$  with which the response is -3 dB. Note that because of the rapid change of attenuation the response is only -0.9 dB at  $1.2f_s$ .

However, it also shows that a true maximally flat characteristic is obtained only if the correct values of box size  $C_{as}$  and especially  $Q_t$  are chosen also. It is easy to show from Eq. (20) that in any alignment, at the upper resonant frequency ( $f_h$  of Fig. 5), the response is

$$E(j\omega) = j(Q_t\omega_h/\omega_s)/[1 - (\omega_b^2/\omega_h^2)] \quad (42)$$

that is, the response varies directly with  $Q_t$ . Also at the box resonant frequency,  $f_b$

$$E(j\omega) = (C_{ab}/C_{as})(\omega_b^2/\omega_s^2) \quad (43)$$

that is, the response is independent of  $Q_t$ . (The response at  $f_l$  is similar to Eq. (42) when  $\omega_h$  is replaced by  $\omega_l$ , but as this is in the attenuation band, it is less important.) Thus if  $Q_t$  is twice the optimum value, there will be a response peak 6 dB high. Now as a general rule a speaker with a  $Q$  of about 0.4, as required in this case, is usually of high quality.

A  $Q$  of 0.8 is typical of a medium quality speaker and a  $Q$  of 1.6 is typical of a low ("popular" or "skimped-magnet") quality speaker. Thus these speakers would

<sup>1</sup> Hence the expression Butterworth box. However, in spite of the phonetic similarity, butter boxes are not in general suitable as loudspeaker enclosures.

have response peaks (at  $1.76\omega_s$  in this case) of 6 dB and 12 dB, respectively, if fed from a zero output impedance amplifier, 12 dB and 18 dB if fed from an amplifier with impedance equal to loudspeaker resistance  $R_e$  (e.g., pentode with 6-dB negative voltage feedback), and even more with higher amplifier impedances. Hence the expression "boom box."

An amplifier with negative output impedance half that of the loudspeaker resistance  $R_e$ , a quite feasible figure, would correct the medium quality speaker, and reduce the peak on the cheaper one to 6 dB. An amplifier with a negative output impedance three quarters of  $R_e$ , to correct the cheaper speaker, is possible but would need care in respect of stability (see Section XII).

### Fifth-Order Butterworth Response

This has the characteristic

$$|E(j\omega)|^2 = 1/[1 + (\omega_o/\omega)^{10}]. \quad (44)$$

The operational form can be factorized to

$$E(p) = 1/[(1 + 1/pT_o)(1 + \sqrt{5}/pT_o + 3/p^2T_o^2 + \sqrt{5}/p^3T_o^3 + 1/p^4T_o^4)] \quad (45)$$

which is the characteristic of two filters in cascade: 1) a first-order filter which can be provided by a CR network with a time constant  $T_o$ , and 2) a fourth-order filter provided by a loudspeaker and box for which

$$T_o = T_s = T_b \quad (46)$$

$$Q_t = 0.447 \quad (47)$$

$$C_{as}/C_{ab} = 1. \quad (48)$$

The alignment, no. 10 of Table I, has the advantage if the extra box size can be tolerated (a smaller value of  $C_{as}/C_{ab}$  means a larger box) that a maximally flat response can be obtained down to the loudspeaker resonant frequency, while at the same time, a very simple "rumble" filter tapers off the input to the amplifier in the attenuation band. This helps the amplifier, but more importantly it greatly reduces the maximum flux density in the output transformer and also the maximum excursion of the loudspeaker (see Section X and Fig. 10).

### Sixth-Order Butterworth Response

This has the characteristic

$$|E(j\omega)|^2 = 1/[1 + (\omega_o/\omega)^{12}] \quad (49)$$

while the operational form may be factorized to

$$E(p) = 1/[(1 + 1.932/pT_o + 1/p^2T_o^2)(1 + 1.414/pT_o + 1/p^2T_o^2)(1 + 0.518/pT_o + 1/p^2T_o^2)]. \quad (50)$$

As in the previous case, the overall alignment is achieved by providing one factor with an external filter, in this case second order, and making the fourth-order response of the loudspeaker plus box the product of the two remaining factors. Thus we can obtain the identical response in three different ways. These are listed in Table I as alignments no. 15, 20, and 26, the three separate classes depending on whether the auxiliary electrical circuit has the lowest, middle, or highest  $x$  value of the three factors in the alignment. Not only do the three alignments produce the same response, but as shown later (Section X and Fig. 10) the cone excursions are identical.

|                       | Alignment Details |                 |      |             | Box Design |           |                 |       | Auxiliary Circuits |           |                |              | Approximately Constant Quantities |                       |
|-----------------------|-------------------|-----------------|------|-------------|------------|-----------|-----------------|-------|--------------------|-----------|----------------|--------------|-----------------------------------|-----------------------|
|                       | No.               | Type            | k    | Ripple (db) | $f_3/f_s$  | $f_3/f_b$ | $C_{as}/C_{ab}$ | $Q_t$ | $f_{aux}/f_3$      | $y_{aux}$ | Peak Lift (db) | $f_{pk}/f_3$ | $\frac{C_{as}f_s^2}{C_{ab}f_3^2}$ | $\frac{Q_t f_b}{f_s}$ |
| Quasi-Third Order     | 1                 | QB <sub>3</sub> | ---  | ---         | 2.68       | 1.34      | 10.48           | .180  | ---                | ---       | ---            | ---          | 1.47                              | .360                  |
|                       | 2                 | QB <sub>3</sub> | ---  | ---         | 2.28       | 1.32      | 7.48            | .209  | ---                | ---       | ---            | ---          | 1.44                              | .362                  |
|                       | 3                 | QB <sub>3</sub> | ---  | ---         | 1.77       | 1.25      | 4.46            | .259  | ---                | ---       | ---            | ---          | 1.43                              | .367                  |
|                       | 4                 | QB <sub>3</sub> | ---  | ---         | 1.45       | 1.18      | 2.95            | .303  | ---                | ---       | ---            | ---          | 1.41                              | .371                  |
| Fourth Order          | 5                 | B <sub>4</sub>  | 1.0  | ---         | 1.000      | 1.000     | 1.414           | .383  | ---                | ---       | ---            | ---          | 1.41                              | .383                  |
|                       | 6                 | C <sub>4</sub>  | .8   | ---         | .867       | .935      | 1.055           | .415  | ---                | ---       | ---            | ---          | 1.41                              | .384                  |
|                       | 7                 | C <sub>4</sub>  | .6   | 0.2         | .729       | .879      | .729            | .466  | ---                | ---       | ---            | ---          | 1.37                              | .386                  |
|                       | 8                 | C <sub>4</sub>  | ---  | 0.9         | .641       | .847      | .559            | .518  | ---                | ---       | ---            | ---          | 1.36                              | .392                  |
|                       | 9                 | C <sub>4</sub>  | ---  | 1.8         | .600       | .838      | .485            | .557  | ---                | ---       | ---            | ---          | 1.35                              | .398                  |
| Fifth Order           | 10                | B <sub>5</sub>  | 1.0  | ---         | 1.000      | 1.000     | 1.000           | .447  | 1.00               | ---       | ---            | ---          | ---                               | ---                   |
|                       | 11                | C <sub>5</sub>  | .7   | ---         | .852       | .934      | .583            | .545  | 1.43               | ---       | ---            | ---          | ---                               | ---                   |
|                       | 12                | C <sub>5</sub>  | .4   | 0.25        | .724       | .889      | .273            | .810  | 2.50               | ---       | ---            | ---          | ---                               | ---                   |
|                       | 13                | C <sub>5</sub>  | .355 | 0.5         | .704       | .882      | .227            | .924  | 2.93               | ---       | ---            | ---          | ---                               | ---                   |
|                       | 14                | C <sub>5</sub>  | .278 | 1.0         | .685       | .877      | .191            | 1.102 | 3.60               | ---       | ---            | ---          | ---                               | ---                   |
| Sixth Order Class I   | 15                | B <sub>6</sub>  | 1.0  | ---         | 1.000      | 1.000     | 2.73            | .299  | 1.00               | -1.732    | 6.0            | 1.07         | ---                               | ---                   |
|                       | 16                | C <sub>6</sub>  | .8   | ---         | .850       | .868      | 2.33            | .317  | 1.01               | -1.824    | 7.7            | 1.06         | ---                               | ---                   |
|                       | 17                | C <sub>6</sub>  | .6   | ---         | .698       | .750      | 1.81            | .348  | 1.02               | -1.899    | 10.1           | 1.05         | ---                               | ---                   |
|                       | 18                | C <sub>6</sub>  | .5   | ---         | .620       | .698      | 1.51            | .371  | 1.03               | -1.930    | 11.6           | 1.05         | ---                               | ---                   |
|                       | 19                | C <sub>6</sub>  | .414 | 0.1         | .554       | .659      | 1.25            | .399  | 1.04               | -1.951    | 13.2           | 1.04         | ---                               | ---                   |
| Sixth Order Class II  | 20                | B <sub>6</sub>  | 1.0  | ---         | 1.000      | 1.000     | 1.000           | .408  | 1.00               | 0         | ---            | ---          | ---                               | ---                   |
|                       | 21                | C <sub>6</sub>  | .8   | ---         | .844       | .954      | .722            | .431  | 1.10               | -.438     | 0.2            | 2.36         | ---                               | ---                   |
|                       | 22                | C <sub>6</sub>  | .6   | ---         | .677       | .917      | .500            | .461  | 1.21               | -.941     | 1.1            | 1.77         | ---                               | ---                   |
|                       | 23                | C <sub>6</sub>  | .5   | ---         | .592       | .902      | .414            | .484  | 1.27               | -1.200    | 1.9            | 1.63         | ---                               | ---                   |
|                       | 24                | C <sub>6</sub>  | .414 | 0.1         | .520       | .890      | .353            | .513  | 1.31               | -1.414    | 3.0            | 1.55         | ---                               | ---                   |
|                       | 25                | C <sub>6</sub>  | .268 | 0.6         | .404       | .876      | .276            | .616  | 1.37               | -1.732    | 6.0            | 1.47         | ---                               | ---                   |
| Sixth Order Class III | 26                | B <sub>6</sub>  | 1.0  | ---         | 1.000      | 1.000     | .732            | .518  | 1.00               | +1.732    | ---            | ---          | ---                               | ---                   |
|                       | 27                | C <sub>6</sub>  | .268 | 0.6         | .778       | .911      | .110            | 1.503 | 2.73               | 0         | ---            | ---          | ---                               | ---                   |
|                       | 28                | QB <sub>3</sub> | ---  | ---         | .952       | .980      | 1.89            | .328  | 1.08 mean          | ---       | 6.0            | 0            | ---                               | ---                   |

Table I. Summary of loudspeaker alignments.

This illustrates a general principle that box size can be exchanged for amplifier power. The only additional penalties are as follows:

- 1) additional heating of the voice coil by signals in the region of the cutoff frequency, and
- 2) the requirement of a smaller value of  $Q_t$  as the box volume is decreased.

The performance required of the auxiliary filtering is given in the last four columns of Table I, whose terms are illustrated in Fig. 6. Instead of the parameter  $x$  in the expression

$$E(p) = 1/(1+x/pT_o+1/p^2T_o^2) \quad (51)$$

the response shapes are defined in Table I by the parameter  $y$  in the expression

$$|E(j\omega)|^2 = 1/[1+y(\omega_o/\omega)^2+(\omega_o/\omega)^4] \quad (52)$$

where

$$y = x^2 - 2 \quad (53)$$

as given in a previous paper [6]. When  $y$  is zero or positive there is no peak in the response as shown in Fig. 6, but when  $y$  is negative there is a peak whose frequency and amplitude are given in Table I. The amplitude of response at the nominal cutoff frequency  $f_{aux}$  of this auxiliary filter is given by

$$|E(j\omega)| = 1/(2+y)^{1/2}. \quad (54)$$

### Chebyshev Responses

If the real values of the poles of a Butterworth function are all multiplied by the same factor  $k$ , which is less than one, a Chebyshev or "equal ripple" function results [7]. Chebyshev filters are characterized by a flat response in the passband except for ripples which are equal in

amplitude, (see curve 8 of Fig. 8). Beyond cutoff, the response falls at a rate whose maximum is greater than the asymptotic slope. Typical values are tabulated in Table I with the type names  $C_4$ ,  $C_5$ , and  $C_6$  representing Chebyshev responses of fourth, fifth, and sixth order. It will be seen from the table that a considerable change in alignment occurs before the ripples become serious in magnitude. For our purpose here, the Chebyshev responses provide a means of carrying the useful response of the speaker plus box combination well below the speaker resonant frequency  $f_s$  (which is also cutoff frequency  $f_o$  in the Butterworth cases). This is done by tuning the box to below  $f_s$ , but not as low as the cutoff frequency (defined here as  $f_3$ , the frequency where the response is 3 dB down). The box size  $C_{ab}$  is increased, and to some extent, so is  $Q_t$ .

The increase in useful low-frequency response is considerable. In alignment no. 9, a response down to  $0.6f_s$  is obtainable without amplifier assistance, if a ripple of 1.8 dB can be tolerated. In alignment no. 25, where a maximum lift of 6 dB is required from the amplifier before its response falls off, a flat response can be obtained down to nearly  $0.4f_s$ .

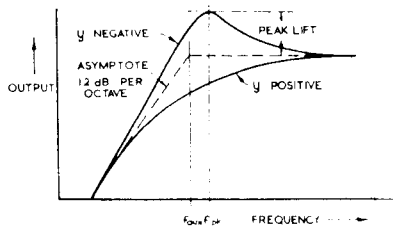


Fig. 6. Typical curves for second-order auxiliary filter, illustrating terms used in Table I.

### Quasi-Butterworth Third-Order Responses

This long name disguises a class of responses characterized by

$$|E(j\omega)|^2 = 1/[1 + y_3(\omega_o/\omega)^6 + y_4(\omega_o/\omega)^8] \quad (55)$$

that is, in the expression for the modulus of the fourth-order filter, there are zero coefficients for the second and fourth powers of frequency, with nonzero coefficients for both the eighth and sixth powers. This type of response yields a series of alignments, nos. 1-4 of Table I, in which the cutoff frequency (again defined here as the frequency  $f_3$  where the response is 3 dB down) is above the speaker resonant frequency. So also is the box resonant frequency, but again, not to the same extent. As the cutoff frequency is made higher, these alignments require smaller box volumes, and lower values of  $Q_t$ .

## VI. GENERAL DISCUSSION OF TABLE I

It will be seen that alignments no. 1-9 provide a means of varying the cutoff frequency of a loudspeaker-box combination over a wide range. The last two columns for these alignments illustrate two interesting properties which remain substantially constant ( $\pm 5\%$ ) over this wide range.

1) The expression  $C_{as}f_s^2/C_{ab}f_3^2$  is substantially constant around 1.41. This means that if a given speaker for

which  $C_{as}$  and  $f_s$  are constant is placed in different boxes to provide different cutoff frequencies, the box volume will vary with inverse frequency squared. This illustrates a fact long known to designers of vented boxes, but rather blurred by the exponents of "revolutionary new concepts," that the bigger the box, the better the low-frequency response. It is also interesting to note that

$$C_{as}f_s^2 = 1/4\pi^2 M_{as} = S_d^2/4\pi^2 M_{ms} \cong 1.41 C_{ab}f_3^2 \quad (56)$$

that is, for a given cutoff frequency of the combination, the box size varies with the square of diaphragm area  $S_d^2$  and inversely with  $M_{ms}$ . In other words, if the mass of the loudspeaker  $M_{ms}$  is fixed and the compliance  $C_{as}$  is varied to give a different resonant frequency  $f_s$ , then the box volume  $C_{ab}$  for a given cutoff frequency  $f_3$  remains substantially constant. To this extent, and also in the expression for efficiency (Eq. (66)) the compliance of the loudspeaker is *unimportant*.

2)  $Q_t f_b/f_s$  lies around 0.38. If Eq. (18) is rewritten as

$$Q_t = \omega_s M_{as}/R_{at} \quad (57)$$

then the expression above becomes  $\omega_b M_{as}/R_{at}$  which can be thought of as the total  $Q$  of the speaker at the box resonant frequency. This remains nearly constant through alignments no. 1-9.

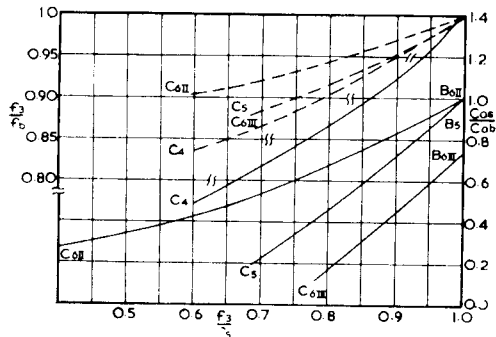
Certain alignments, no. 13, 14, and 27 with no. 12 as a borderline case, which require auxiliary filtering with large attenuation at the cutoff frequency of the whole system, must be considered suspect, since they postulate high acoustic efficiencies in the region of cutoff. Remember that the basis of the theory is that the overall efficiency is low. In the borderline case, no. 12 for example, the peak efficiency will be just above cutoff frequency and will be approximately  $2.5^2$  times the loudspeaker efficiency. If the loudspeaker is 4% efficient, this means a maximum overall efficiency of 25%. Around this point, the basic assumptions will become inaccurate, especially if resistive losses in the box are large.

Similarly, for reasons of cone excursion (considered in Section X), alignments with smaller values of  $f_3/f_b$  such as nos. 17-19 should be avoided if possible. These particular alignments which do give good low-frequency responses in small box volumes would probably be unpopular anyway since they make such great demands on amplifier output in the region of cutoff.

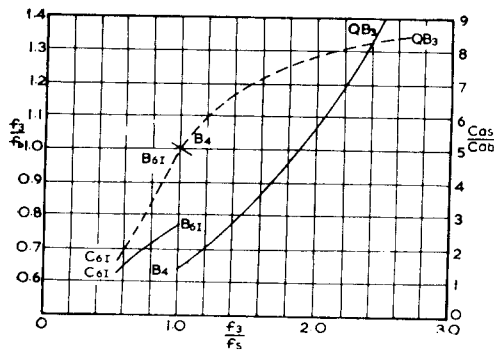
Alignment no. 28 is interesting in that it represents the result of "pure" bass lift. In the other alignments which use "amplifier aiding," the response often rises near cutoff, but always falls off ultimately at lower frequencies at a rate of 6 or 12 dB per octave. In this way, although increased amplifier output may be required over a comparatively narrow range of frequencies, a greatly decreased output, and with it, a greatly decreased cone excursion, is required at the lower frequencies. But in alignment no. 28, a simple low-frequency lift of 6 dB, such as results from a network with two resistors and a capacitor, is required. The mean frequency of lift (at which the lift is 3 dB) is  $1.08f_3$ . However, since the maximum lift continues to the lowest frequencies, the amplifier would be more likely to cause intermodulation distortion with "rumble" components. However it does give some decrease of box volume compared with alignment no. 5.

It should be emphasized that these alignments are by no means the only ones possible. They have been chosen as the ones most likely to be useful and as showing the





a



b

Fig. 7.  $f_3/f_b$  (dashed curves) and  $C_{as}/C_{ab}$  (solid curves) versus  $f_s/f_s$ . a. For design of medium and large boxes; alignment types  $B_1-C_1$ ,  $B_2-C_2$ , and  $B_3-C_3$  class II and III. b. For design of small boxes; alignment types  $QB_3-B_4$  and  $B_5-C_6$  class I.

trend of results. If more sophisticated filtering in the amplifier is possible, the choice widens greatly, e.g., there are six alignments for the eighth-order Butterworth response, each with its fourth-order amplifier filter and the ratios  $C_{as}/C_{ab}$  of 0.518, 0.681, 1.000, 1.316, 1.932, and 2.543.

Another possibility would be the use, instead of the "quasi-Butterworth" responses, of "sub-Chebyshev" responses, i.e., response functions derived by multiplying the real coordinates of the Butterworth poles by a constant  $k$  which is greater than 1.

In answer to the question proposed in 1) of Section I—What is a large box?—it would appear that a medium sized box would be one for which  $V_b$  is about the same value as  $V_{as}$ , say  $C_{as}/C_{ab}$  lies between 1 and 1.414. For large boxes,  $C_{as}/C_{ab}$  is less than 1, for small boxes  $C_{as}/C_{ab}$  is greater than 1.414. Table I shows that smaller boxes demand a smaller value of  $Q_t$ . Thus if  $Q_t$  is not properly controlled, the smaller boxes will tend to cause a greater peak at  $f_b$ , while larger boxes will cause the peak to diminish. Fig. 7 is plotted from the points of Table I. Typical response curves for alignments no. 3, 5, and 8 are given in Fig. 8.

## VII. TO DESIGN A BOX FOR A GIVEN LOUDSPEAKER

First, the following three loudspeaker parameters must be known: 1) the resonant frequency  $f_s$ , 2) the  $Q$  values  $Q_a$  and  $Q_b$ , the latter being usually the controlling factor. This is discussed in more detail in Section IX, Eqs. (71) and (72), and 3) the acoustic compliance  $C_{as}$ . This is expressed most conveniently as  $V_{as}$ , the volume of air

whose acoustic compliance is equal to that of the speaker.

Since in general the acoustic compliance, from [3, Eq. (5.38)] is given by

$$C = V/\rho_0 c^2 \quad (58)$$

then

$$C_{as}/C_{ab} = V_{as}/V_b \quad (59)$$

where  $V_b$  is the volume of the box.

The design is commenced in one of two ways:

1) If the box size is limited,  $V_b$  is taken as the assigned value. Remember this is the net volume, and that the bracing and the volume displaced by the loudspeaker and the vent (say 10%) must be subtracted from the gross volume. From this value and the known value of  $V_{as}$ , the ratio  $C_{as}/C_{ab}$  is found, and thence either from Fig. 7 or interpolation from Table I, the values of  $f_3/f_s$ ,  $f_3/f_b$ , and  $Q_t$ . Hence  $f_3$  and  $f_b$  are found.

2) If a certain frequency response is required, then  $f_3$  is the starting point. The ratio  $f_3/f_s$  is found, then from Fig. 7, or by interpolation from Table I,  $f_3/f_b$ ,  $C_{as}/C_{ab}$ , and  $Q_t$ . Hence  $f_b$  and  $V_b$  are found.

The choice of alignment will depend largely on what can be done with the amplifier circuits. For a straightforward amplifier with no filtering, alignments no. 1–9 would be chosen. If a slightly larger box is possible, alignments no. 10 and 11, with their simple CR input filtering make it possible to ease the power handling requirements of both speaker and amplifier. If a more sophisticated design of input filtering is possible as described in Sections V and XII, alignments 15–17 can be used to obtain good acoustic output from small boxes at the expense of higher electrical power output from the amplifier, while alignments no. 20–25 are the most suitable if a fair sized box is available and only moderate lift is required from the amplifier, although in all the fifth- and sixth-order cases, the power required from the amplifier and the excursion demanded of the speaker decrease rapidly below cutoff.

Having found  $f_b$  and  $V_b$ , the vent dimensions may be found using the methods of the standard texts [8]. However, the following adaptation of the method has proven useful for calculation. The standard form is

$$V_b = 1.84 \times 10^8 S_v / \omega_b^2 L_v \quad (60)$$

where  $S_v$  is the cross-sectional area of the vent, in square inches, and  $L_v$  is the effective length of the vent, in inches, which includes its actual length together with an end correction.

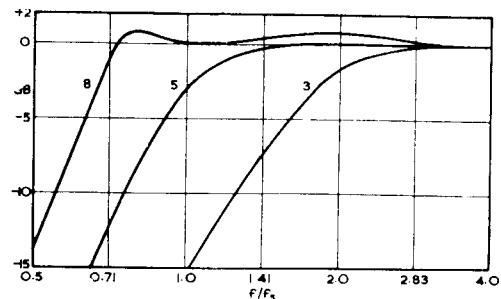


Fig. 8. Typical response curves for identical loudspeakers, but different box sizes.  $C_{as}/C_{ab} = 0.56, 1.41, \text{ and } 4.46$ , corresponding to alignments no. 8, 5, and 3 (types  $C_4, B_4, \text{ and } QB_3$ ) of Table I.

This is written more conveniently as

$$L_v/S_v = 1.84 \times 10^8 / \omega_b^2 V_b. \quad (61)$$

The quantity  $L_v/S_v$ , which has the dimension of inches<sup>-1</sup>, is equivalent to an inductance (acoustic mass) which resonates at  $\omega_b$  with a capacitance (acoustic compliance) equivalent to  $V_b$ . When  $L_v/S_v$  is found, a value is chosen for the vent area  $S_v$ . It has been shown already in connection with Eq. (6) that the radiation resistance, and therefore the operation of the vented box, is independent of the value of  $S_v$ . Now it is usually stated that  $S_v$  should normally be the same as the effective radiating area of the cone [8], i.e.,  $S_d$ . However, this will often involve an excessive length of vent, especially in small boxes and at low cutoff frequencies, because, since  $L_v/S_v$  is fixed, the volume  $L_v S_v$  displaced by the vent varies as  $S_v^2$ . At the same time, a small amount of distortion is generated in the vent (see [4, Eq. 6.33]) which is a maximum near the box resonant frequency  $\omega_b$  and is proportional to  $L_v$ . On the other hand, Novak [2] quotes 4 in<sup>2</sup> as the lower unit.<sup>2</sup> As shown before, a small area vent has still a high value of  $Q$ . However, it will also have higher alternating velocities of air, and this will limit the amount of acoustic power that can be handled linearly. The only advice that can be given is to design the vent area as large as possible in the particular circumstances, up to a limit equal to the piston area.

The maximum length of  $L_v$  is usually quoted as  $\lambda/12$  where  $\lambda$  is the wavelength of sound at the loudspeaker resonant frequency  $f_s$ . The actual requirement is that the vent, which is essentially a transmission line, should look like a lumped constant mass at all the frequencies for which the box is effective. That is, it must still be rather shorter than  $\lambda/4$  at frequencies somewhat above  $f_h$  of Fig. 5. The value of  $f_h$  with respect to  $f_s$  will depend on the box tuning. But it also varies with  $C_{as}/C_{ab}$ ; with a smaller box,  $f_h$  is higher.

With the chosen area of vent, first calculate the part of  $L_v/S_v$  due to the end correction. This length  $L''$  is usually quoted as

$$L'' = 1.70R \quad (62)$$

where  $R$  is the effective radius of the vent, i.e.,

$$(L_v/S_v)_{end} = 0.958/\sqrt{S_v}. \quad (63)$$

This applies to pipes with both ends flanged. When a free-standing pipe is used, the end correction is

$$L'' = 1.46R \quad (64)$$

and

$$(L_v/S_v)_{end} = 0.823/\sqrt{S_v}. \quad (65)$$

In a pipe the end correction is not usually a large part of  $L_v/S_v$ . It forms the larger part when the vent is a simple hole in the front panel and then Eq. (63) is correct.

A method favored by the writer, if styling permits, is to build a shelf into the bottom of the box as in Fig. 9, with a spacing  $l$  from the back panel equal to the height

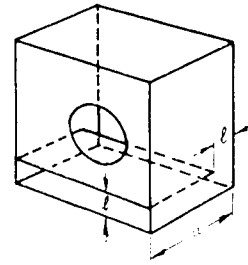


Fig. 9. Simple method of making a tunnel or duct.

of the opening in the front panel. In this case, the effective length of the tunnel is the box depth  $d$  plus the end correction as given by Eq. (62) and allowances for thickness of lumber. This vent is tuned by varying  $l$ .

When  $(L_v/S_v)_{end}$  is found, it is subtracted from the required value of  $L_v/S_v$ , and from this, the actual length  $L_v'$  is calculated. If this value is unsuitable, another value of  $S_v$  is tried and so on (see Appendix).

With regard to box dimensions, it is desirable to take all precautions to prevent strong standing waves. If a corner box is made, the problem is usually fairly easy to solve since the box sides are splayed at least in two dimensions. If a rectangular box is made, and if styling allows, the inside dimensions should be in the preferred ratio for small rooms, that is, 0.8:1.0:1.25 or 0.6:1.0:1.6. In any case, the speaker should be mounted away from the center of the front panel.

The need for sound sealing, with good glued joints, adequate bracing, and adequate damping of the internal surfaces has been stressed often before, so no more need be said of it here. The same is true for the improvement in performance that is obtained by placing the box in the corner of the room, and also by building the sides of the box right down to the floor. However, this last does not seem to be realized sufficiently and the current fad for mounting all furniture on legs causes much unnecessary loss of performance in loudspeaker boxes.

Finally the value of  $Q_t$  required by the alignment is compared with the values  $Q_a$  and  $Q_e$  available, and suitable adjustments are made to the amplifier to achieve a correct overall  $Q_t$ . This is dealt with in Section XII, and a worked example is given in the Appendix.

## REFERENCES

1. E. J. Jordan, "Loudspeaker Enclosure Design," *Wireless World*, vol. 62, pp. 8-14 (Jan. 1956); pp. 75-79 (Feb. 1956).
2. J. F. Novak, "Performance of Enclosures for Low-Resonance High-Compliance Loudspeakers," *IRE Trans Audio*, vol. AU-7, pp. 5-13 (Jan.-Feb. 1959).
3. L. L. Beranek, *Acoustics* (McGraw-Hill, London, 1959).
4. H. F. Olson, *Elements of Acoustical Engineering*, 2nd ed. (Van Nostrand, Princeton, N.J., 1947), pp. 154-156.
5. O. W. Eshbach, Ed., *Handbook of Engineering Fundamentals* (John Wiley, New York, 1936), pp. 8-60.
6. A. N. Thiele, "Television IF Amplifiers with Linear Phase Response," *Proc. IRE (Aust.)*, vol. 19, p. 655 (Nov. 1958).
7. J. L. Stewart, *Circuit Theory and Design* (John Wiley, New York, 1956), pp. 159-163.
8. F. Langford-Smith, *Radiotron Designer's Handbook*, 4th ed. (A. W. V. Co., Sydney, 1952), p. 847.

<sup>2</sup> This is presumably for the particular case he considers where  $f_b$  is 25 Hz, and the acoustic output power is high. For a higher box resonant frequency and/or lower power, an even smaller vent area seems permissible.

## THE AUTHOR



Neville Thiele was educated at the University of Queensland and the University of Sydney, graduating as Bachelor of Engineering in 1952. He joined the staff of E.M.I. Australia Ltd. in 1952 as a development engineer in the Special Products Division. During 1955 he spent six months in England, Europe and the United States and on return was responsible for the develop-

ment of E.M.I. Australia's television receiver. He was appointed advance development engineer in 1957.

Mr. Thiele joined the Australian Broadcasting Commission in 1962, and is now senior engineer, design and development, responsible for the Federal Engineering Laboratory, and with design investigation of equipment and systems for sound and vision broadcasting.